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THE STABILITY OF A DRY PATCH ON A WETTED WALL

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Abstract—A theoretical investigation is described into the flow of a liquid film around a stable dry patch on a wetted inclined wall. Observations have suggested the following model. The flow is assumed to be undisturbed by the dry patch except in a thin region around its boundary, called the collar, which has some features of a boundary layer. This is patched to the main flow of the film by conservation arguments. It is possible to calculate the maximum film thickness for which such a configuration is possible, and also the dimensions of the collar and the dry patch in these cases. The results are compared with previous investigations.

NOMENCLATURE

$0xyz$, Cartesian coordinates;
u, v, w, corresponding velocity components;
ρ, density of liquid;
ν, kinematic viscosity of liquid;
g, gravitation constant;
h, film thickness;
T, surface tension;
γ, angle of contact;
a, c, d, A, collar cross-section parameters: radius, thickness, width, area;
U, tangential velocity in collar;
$\bar{U}$, mean value of U over cross-section;
s, arc length measured along dry-patch boundary;
θ, angle between Ox and tangent to dry-patch boundary;
κ, curvature of free surface;
R, radius of curvature of dry-patch boundary;
R_0, value of R at apex;
p, pressure;
l, length scale;
k, L, M, N, P, K, dummy constants;
η, a/h;
α, inclination of plane to vertical;
H, D, thickness and width of fingers far downstream.

1. INTRODUCTION

It is a familiar observation that when a film of water runs down an inclined surface it tends to do so not in a more or less uniform sheet but in a number of trickles or fingers with dry areas in between, which are often in the approximate form of parabolas with vertex upwards. This phenomenon is of great importance in chemical engineering in any situation where gas and liquid are to be brought into contact, such as in distillation columns and heat exchangers. Film rupture is undesirable because it may lead to reduced efficiency and overheating or corrosion of the dry area. One would like to estimate, then, the minimum thickness at which the film will remain intact and the area of the dry patches when they are present.

This has been the subject of a number of investigations and one may cite the work of Ponter *et al.* [1] and Hartley and Murgatroyd [2] (hereafter referred to as [1] and [2] respectively). In [1] a good deal of literature is considered and some interesting experiments are reported. The qualitative aspects of the situation are as expected. When the film is sufficiently thick dry patches do not appear; when it is sufficiently thin, stable (i.e. permanent) dry patches appear. In between there is a regime in which the film ruptures from time to time to form a dry patch which is subsequently filled in or swept away.

In both [1] and [2] simple theoretical models of the flow are analysed and compared with experiment, with varying success. The object of the present work is to present a considerably more detailed theoretical study in an attempt to improve our understanding of the hydrodynamical processes.

2. APPROXIMATIONS

Although in most practical and experimental situations the film is flowing along a circular tube (either outside or inside) the present theory will describe flow on an inclined (usually vertical) plane. This approximation (which is used in all other theoretical attempts) will be adequate when the tube radius is large enough compared with other lengths; typically the tube radius is 2–3 cm and the film thickness is about 10^{-2} cm.

We consider the steady flow in the presence of a stable dry patch and the configuration is as shown in Fig. 1. Cartesian axes $0xyz$ are fixed with 0 in the plane.

The theories presented in I and II were based on the assumption that the flow is everywhere parallel to the solid boundary, and then (referring to Fig. 2) equating the downward momentum flux to the upward force due to surface tension along CA leads to

$$\frac{2}{15}\frac{\rho g^2 h^5}{\nu^2} = T(1-\cos\gamma) \tag{1}$$

when the plane is vertical. (This calculation is carried out incorrectly in [2]; there is an erroneous factor of $\frac{1}{2}$ in equation (4).)

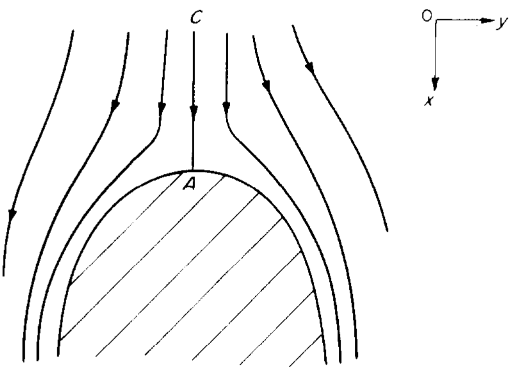

FIG. 1. Sketch of flow round a dry patch (shaded area).

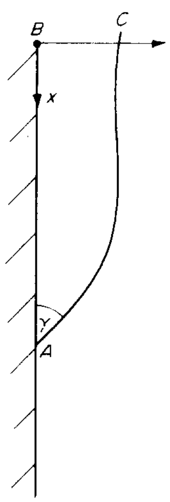

FIG. 2. Cross-section of film through apex A.

Although heat and mass transfer are supposed to be taking place no account of them is taken in this simple theory. McPherson [3] investigated some of these effects and the conclusion seems to be that their *direct* effects are negligible. (We should point out that McPherson deals with a horizontal film which is dragged along by a stream of gas flowing over it; this force plays the same role as gravity in the present theory.) It seems reasonable to assume, then, that temperature gradients, evaporation, and so on, will enter only *parametrically* into the problem, that is, they will determine the viscosity, surface tension and angle of contact, but will not produce forces directly. This assumption is implicit in [1] and [2] and will probably be adequate unless the temperature gradients are so large as to produce turbulence or boiling.

The aim of the present work is to present criticisms of equation (1) of a more fundamental nature and to present an improved theory which meets these criticisms. First we may notice that in considering the force balance along CA no account is taken of gravity or viscous drag on the wall; of course these will be equal and opposite over much of the film, well upstream of A, but this cannot be the case near the edge of the film where the fluid is slowed down.

Secondly, it appears from equation (1) that only one value of h is possible for each value of γ; and while γ may vary somewhat from its static value, it is known from experiments that a stable dry patch can exist for *all* h up to a certain maximum value.

It has been noticed by several workers that the film thickens near the boundary of the dry patch to form a collar; this is clearly visible in the photographs in [1] and is discussed by McPherson, although he is unable to predict its dimensions. This observation provides the key to the present theory. It is proposed that well away from the dry patch the balance of forces is between gravity and viscous drag. When the fluid enters the collar the radial balance (i.e. parallel to the plane and normal to the dry patch boundary) will be hydrostatic; the tangential balance is more complicated and will be discussed later. The collar thus resembles a boundary-layer, in which surface tension forces take over from viscous drag.

The theory given here will be partly empirical in that no attempt will be made to predict the shape of the collar cross-section; this will be assumed to be an arc of a circle making the appropriate angle with the solid boundary. It will be possible however to find the radius of this circle and also the shape of the dry-patch boundary, near the apex A. The two regions, film and collar, will be simply patched together using conservation of mass and momentum.

Downstream of the apex the flow will develop into fingers (assuming that several dry patches are present) and the approximations described above will break down as the distinction between the main film and the collar becomes blurred. Ultimately the streamlines will be straight and this leads to a fairly tractable problem which is dealt with in Chapter 5.

3. FLOW NEAR THE APEX

If the film is uniform far upstream and the plate is vertical we have the velocity components

$$u = \frac{1}{2}\frac{g}{\nu}(z^2 - 2hz) \qquad v = w = 0 \tag{2}$$

and so the volume flux per unit width is $(1/3)(gh^3/\nu)$ and the momentum flux per unit width is $(2/15)(g/\nu)^2 h^5$.

Now consider a typical station on the collar near the apex (Fig. 3). We shall obtain equations expressing the

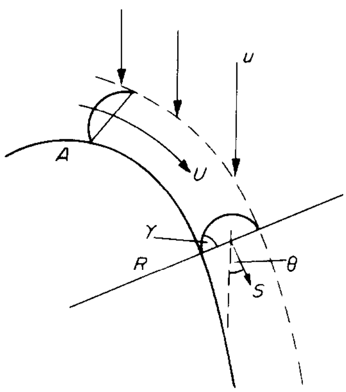

FIG. 3. Sketch of collar and co-ordinate system.

conservation of mass and momentum as fluid enters the collar from the film and flows along it. Arc length measured tangentially along the collar (i.e. parallel to the dry patch boundary) is denoted by s. We assume that the radius of the cross-section of the collar, a, varies only slowly with s, and that the velocity U in the collar is approximately tangential. If $\bar{U}$ is the average value of U over the cross-section and A is the cross-section area, so that $A = a^2(\gamma - \frac{1}{2}\sin 2\gamma)$, conservation of mass gives

$$A\bar{U} = \frac{1}{3}\frac{g}{\nu}h^3 y. \tag{3}$$

The r.h.s. is the total volume flux entering the collar between the apex and the station in question.

We now turn to the balance of tangential momentum. At any point in the fluid in the collar the tangential force due to viscosity is proportional to $\nabla^2 U$, ∇^2 here representing a two-dimensional operator in the cross-section plane ($\partial^2/\partial s^2$ being assumed small in comparison). The tangential component of gravity is $g\cos\theta$.

The momentum entering the collar between two stations a distance ds apart is

$$\frac{2}{15}\left(\frac{g}{\nu}\right)^2 h^5\, \mathrm{d}s \sin\theta$$

and the tangential component of this is

$$\frac{2}{15}\left(\frac{g}{\nu}\right)^2 h^5\, \mathrm{d}s \sin\theta\cos\theta.$$

Averaging this over the cross-section we find that the force produced (which is proportional to the momentum injected per unit arc length) is proportional to

$$\frac{2}{15}\left(\frac{g}{\nu}\right)^2 \frac{h^5}{A}\sin\theta\cos\theta.$$

Finally we must include in the tangential equation of motion the effect of the tangential acceleration produced by mass injection. Since fluid is entering the collar at each station and we have assumed that the cross-section is varying only slowly, it follows that U must depend on s and clearly

$$\frac{\partial\bar{U}}{\partial s} = \frac{1}{3}\frac{g}{\nu}\frac{h^3}{A}\sin\theta.$$

We are thus compelled to account for the non-linear inertia terms in the tangential equation of motion. Under the present approximations these reduce to the single term $U(\partial U/\partial s)$ and we shall replace this by an averaged linear expression

$$U\frac{\partial\bar{U}}{\partial s} = \frac{1}{3}\frac{g}{\nu}\frac{h^3}{A}\sin\theta U.$$

These four terms may now be combined to give the tangential momentum equation

$$\nu\nabla^2 U = -\left[g\cos\theta + \frac{2}{15}\left(\frac{g}{\nu}\right)^2\frac{h^5}{A}\sin\theta\cos\theta\right] + \frac{1}{3}\frac{g}{\nu}\frac{h^3}{A}\sin\theta U. \tag{4}$$

Rewriting this as

$$\nabla^2 U = -M + N^2 U \tag{4a}$$

we obtain

$$\bar{U} = \frac{M}{N^2}[1 - f(Nd)] \tag{5}$$

where $d = 2a\sin\gamma$ is the collar width and f is the solution of a certain canonical boundary value problem described in the Appendix.

The third equation will now be obtained, representing the radial (or normal) balance of momentum. Consider a point where the free surface meets the solid boundary. A balance must be achieved between the force due to surface tension, the hydrostatic force due to the weight of liquid in the collar, and the radial injection of momentum.

The total curvature κ at the point where the free surface meets the solid boundary is given by

$$\kappa = \frac{1}{a} - \frac{\sin\gamma}{R}. \tag{6}$$

This is because we can regard the free surface at any station as (approximately) the surface swept out by the local cross-section moving in a circle of radius R. The appropriate formula for κ may be found in Weatherburn [5].

Resolving the weight of liquid in the collar radially we obtain $2\rho g d \sin\theta$. To this must be added a pressure p_0 which arises from radial momentum injection. Considering again a short arc length ds we see that p_0 is given by

$$\frac{p_0}{\rho} c\,\mathrm{d}s = \frac{2}{15}\left(\frac{g}{\nu}\right)^2 h^5 \sin^2\theta\,\mathrm{d}s \tag{7}$$

c being the collar thickness. (Centrifugal forces have been neglected.) The equation of radial balance is thus

$$T\left(\frac{1}{a} - \frac{\sin\gamma}{R}\right) = \frac{2}{15}\left(\frac{g}{\nu}\right)^2 \frac{\rho h^5}{c}\sin^2\theta + 2\rho g d\sin\theta. \tag{8}$$

The equations (3), (4) and (8) are sufficient when f is known. Equations (3) and (5) yield an equation for y in terms of θ which may be integrated to give the shape of the dry patch boundary. In particular it is easy to obtain the radius of curvature at the apex, R_0, and substitute into (8) with $\theta = \pi/2$, thus determining a. Before displaying these results it will be convenient to transfer to dimensionless variables. We introduce a length l defined by

$$l^{-5} = \frac{2}{15}\left(\frac{g}{\nu}\right)^2 \frac{\rho}{T} \tag{9}$$

and l is about $3{\cdot}5 \times 10^{-2}$ cm for water at 20°C. We take

$$1 - f(t) = \frac{k^2 t^2}{4(1+\frac{1}{2}kt)^2} \tag{10}$$

where $k = 2(\gamma - \frac{1}{2}\sin 2\gamma)/\sin^2\gamma$ as indicated in the Appendix. Then (3) and (5) give

$$y = \frac{3k^2\sin^2\gamma(\gamma - \frac{1}{2}\sin 2\gamma)\dfrac{a^{*4}}{h^{*3}}\left(1 + L\dfrac{h^{*5}}{a^{*2}}\sin\theta\right)\cos\theta}{4[1 + P\sqrt{(h^{*3}\sin\theta)}]^2} \tag{11}$$

where the asterisk denotes a dimensionless form and the constants L and P are given by

$$L = \frac{2}{15}\frac{g}{\nu^2}\frac{l^3}{(\gamma - \frac{1}{2}\sin 2\gamma)}$$

$$P = \frac{5}{8}k^2\sin^2\gamma L.$$

The general form of (11) is $y = F(\theta)$, and this together with the equation dy/dx = tan θ enables us to obtain the shape of the dry patch boundary. In particular it is readily shown that the radius of curvature at the apex is $F'(\pi/2)$, that is

$$R_0 = \frac{\frac{3}{4}k^2\sin^2\gamma(\gamma - \frac{1}{2}\sin 2\gamma)\dfrac{a^{*4}}{h^{*3}}\left(1 + L\dfrac{h^{*5}}{a^{*2}}\right)}{(1 + Ph^{*3/2})^2}. \tag{12}$$

Away from the apex the distinction between collar and film becomes less clear and so the approximations of this section will break down; for this reason the question of the shape of the dry patch boundary will not be taken further. The value of R_0 does however provide some estimate of the size of the dry area.

Substituting (12) into (7) with $\theta = \pi/2$ will yield an equation to determine a^* in terms of h^*. Writing $\eta = a^*/h^*$ this becomes

$$\frac{k^2\sin\gamma(\gamma - \frac{1}{2}\sin 2\gamma)}{1-\cos\gamma}\{Kh^{*2}\eta^2 + h^{*5} - 1 + \cos\gamma\} \times (\eta^3 + Lh^{*3}\eta) + (1 + Ph^{*3/2})^2 = 0 \tag{13}$$

where $K = 2\rho g l^2 \sin\gamma(1-\cos\gamma)/T$. The solution of this to give η in terms of h^* is done numerically but certain results may be obtained by inspection. We observe that when $h^* \to 0$ there is a real root for η given by

$$\tfrac{3}{4}k^2\sin\gamma(\gamma - \tfrac{1}{2}\sin 2\gamma)\eta^3 = 1$$

but if $h^{*5} > 1 - \cos\gamma$ there is no real root because every term in (13) is positive. We thus define a critical value of h^*, h^*_{cr} say, above which no solution is possible and this is the maximum film thickness for which a stable dry patch is possible.

The result $h^{*5}_{cr} = 1 - \cos\gamma$ was obtained in [1] essentially by guessing that the important terms in (7) are T/a and p_0, and in many cases it turns out that this is not far wrong.

In summary, for each value of h^* below the critical value, (13) may be solved to give η and hence a^* and the collar width and thickness, d and c respectively. The value of a^* thus obtained may be substituted into (12) to give R_0. Examples of this are given in the next section.

4. RESULTS

Various quantities of interest are shown graphically in Figs. 4–7. Figure 4 shows the collar thickness $c^* = a^*(1-\cos\gamma)$ and the critical film thickness h^*_c as calculated from equation (1) and the present theory, against γ. The values chosen for T and ν are those for water–air at 20°C. It will be seen that [1] gives results about 10 per cent higher than the present theory over most of the range. Figure 5 shows the radius of curvature at the apex R_0 when $h^* = h^*_c$, against γ. The curvature $\kappa_0 = R_0^{-1}$ is also shown and it appears to vary approximately linearly with γ over much of the range.

The variation of c^* and R^* with h^* as h^* increases

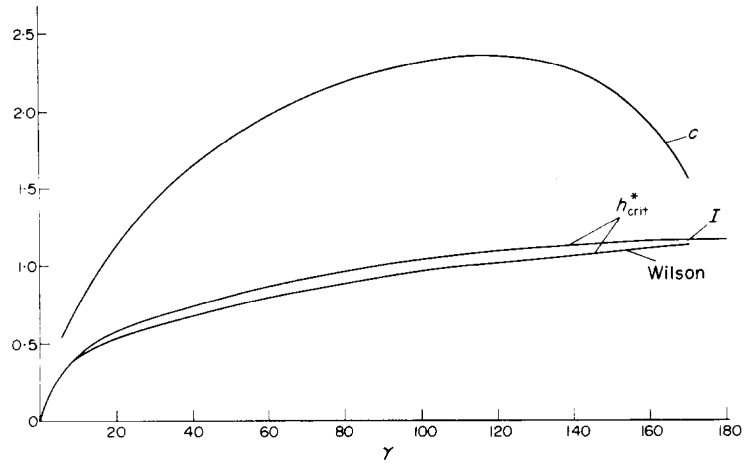

FIG. 4. Graphs of critical film thickness h_{crit} and corresponding collar thickness c against γ. Of the 2 curves for h_{crit} the upper represents the solution of equation (1) and the lower the solution of (12).

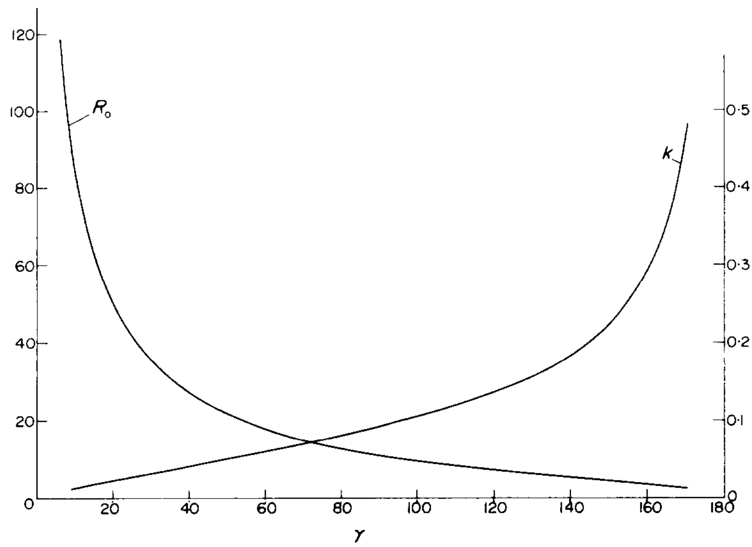

FIG. 5. Graphs of R_0 (radius of curvature of dry patch boundary at apex) and R_0^{-1} at the critical value of h against γ. The left-hand scale refers to R_0, the right-hand scale to R_0^{-1}.

towards h_c^*, for given γ, is shown in Figs. 6 and 7 and it is apparent that they increase rapidly to their critical values as h^* approaches h_c^*.

Unfortunately, although a great deal of experimental work has been reported, a proper comparison is not possible. The most important reason is lack of information about the contact angle. In fact [1] appears to be the only work in which the vital role of this angle is appreciated and it should be emphasised that it must be measured under the appropriate conditions of surface roughness, evaporation and so on. The contact angle is extremely sensitive to these influences as shown in [1]. Furthermore, only the static contact angle has been measured and it is known that the actual angle under dynamic conditions (such as at A) may differ by 20° or more. (Dr. Davies has indicated privately that the authors of [1] were well aware of this point but that it proved impossible to measure this angle in practice.)

The reason why the contact angle has not usually been measured is probably bound up with the source of a further difficulty, namely, that what experimenters have reported is invariably the minimum wetting rate,

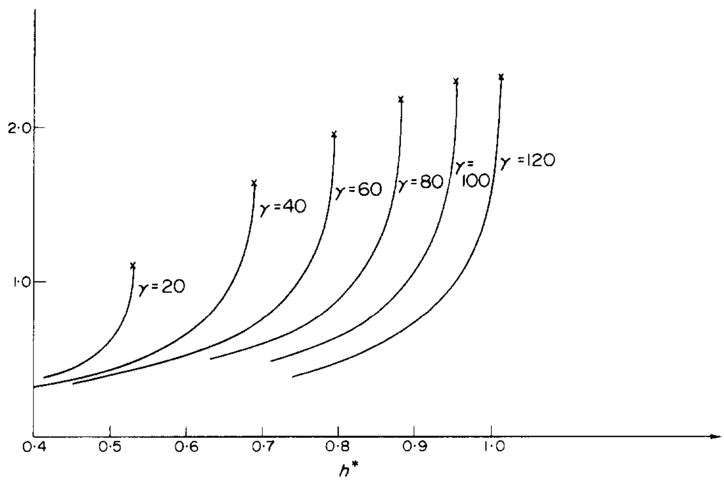

FIG. 6. Graphs of collar thickness c against h for various values of γ. Each curve terminates at the point marked by $\times$, at which $h = h_{crit}$ for that value of γ.

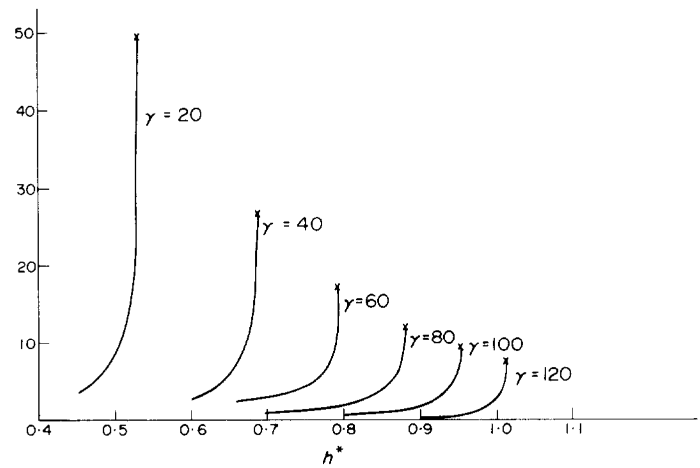

FIG. 7. Graphs of R_0 against h for various values of γ. As in Fig. 6, $\times$ denotes a critical value.

i.e. the least flow rate at which dry patches never appear. What the theory gives is of course the maximum flow rate for which dry patches are possible and in between is the unsteady transition regime described earlier. Since, then, in most measurements the dry patches are absent it seems natural to suppose that the contact angle is unimportant.

However, the major conclusion of this paper is that the surface tension and contact angle are the vital quantities and that errors in measuring them may account for most of the discrepancies between theory and experiment. The extraordinary sensitivity of the contact angle, in particular, to evaporation, absorption and so on, and the difficulty of controlling surface conditions in practical situations, are unfortunate facts which must be faced.

Finally, the whole flow of an intact film may be dynamically unstable. The growth of wavy disturbances has been the subject of many investigations, and there is also instability of flow down a vertical tube to small deviations of the tube from the vertical (Ponter and Davies [4]). These instabilities may cause local thinning of the film leading to rupture, even though the average thickness (which is what is measured, by means of the total flow rate) may be above the expected critical value. Generally it seems to be almost impossible to maintain an intact film on a tube more than about 2 m long. In [1] elaborate precautions were taken to ensure that the tube was straight, circular and vertical and that the film was put on evenly at the top, but nevertheless the results show almost without exception that the film broke down on long (120 cm) tubes more readily than on short (30 cm) ones. This suggests that instabilities are present but have not had a chance to grow on the shorter tubes. We may conclude that it is the results from the shorter tubes which are more reliable because the actual film thickness is more likely to be close to the average.

I should like to suggest that if further experiments are carried out measurements should be taken below the critical wetting rate (as well as above it) to compare with Figs. 6 and 7, and that close attention should be given to the problem of measuring the contact angle under dynamic conditions. The boundary-layer character of the flow suggested here could be investigated by introducing dye streaks so as to pass close to A. The present theory predicts that most of the fluid enters the collar and is not merely swept round it.

5. FLOW FAR DOWNSTREAM

Well downstream of the apex of the dry patch the fluid will flow straight down the plane in a number of fingers and it is interesting to find the shape of the cross-section of these fingers in terms of the volume of fluid flowing in them. We shall assume here that the plane is inclined at an angle α to the vertical.

This was studied in [2] where it was proposed that the cross-section will be such as to minimise the rate of flow of the sum of surface and kinetic energies. One certainly has the feeling that something is minimised but we shall see that this idea is incorrect. Figure 8

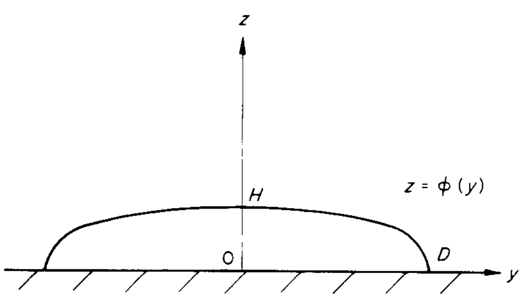

FIG. 8. Cross-section of the finger for downstream.

shows the cross-section of the finger. The streamlines are straight and the cross-section is independent of x (by assumption) and so the velocity field must take the form $[u(y,z),0,0]$. The equations of motion then reduce to

$$\nu\nabla^2 u = -g\cos\alpha + \frac{1}{\rho}\frac{\partial p}{\partial x} \tag{14}$$

in the x direction, and hydrostatic equilibrium in the

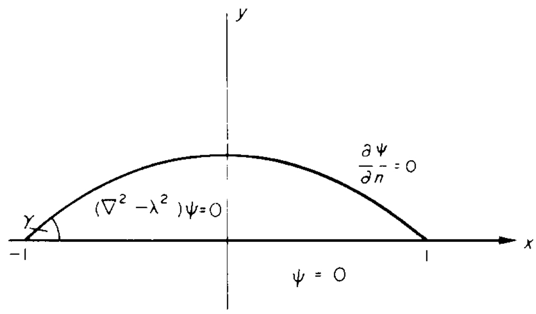

FIG. 9. The canonical boundary value problem.

y and z directions, that is $\partial p/\partial y = 0$ and

$$p = p_1 - \rho g\sin\alpha z \tag{15}$$

where p_1 is the pressure on the plane. There is no normal viscous stress on the free surface and so the free surface condition is

$$p + T\kappa = 0 \quad \text{on} \quad z = \phi(y). \tag{16}$$

Now we observe that κ is independent of x and so $\partial p/\partial x = 0$, and (13) may be reduced to

$$\nabla^2 u = -\frac{g}{\nu}\cos\alpha. \tag{14a}$$

The boundary conditions on the velocity u are

$$\begin{aligned} u &= 0 \quad \text{on} \quad z = 0 \\ \frac{\partial u}{\partial n} &= 0 \quad \text{on} \quad z = \phi(y) \end{aligned} \tag{17}$$

where $\partial/\partial n$ denotes differentiation along the normal. This last condition is that of zero shear stress on the free surface.

It is now possible to see that the problem may be divided into two parts. The cross-section may be determined from (14) and (15), which may be combined to give

$$p_1 - \rho g\sin\alpha\phi + \frac{T\phi''}{(1+\phi'^2)^{3/2}} = 0 \tag{18}$$

the solution of which is the well-known curve called the elastica. (The reason that this is possible is that the velocity does not occur in the specification of the problem to determine ϕ, which is therefore the same as if the fluid were at rest.) Once ϕ has been determined the solution may be inserted into (16) which, together with (13a), will determine u. This problem is intractable in general because of the complicated nature of the function ϕ.

Equation (18) has also been studied in connection with the sessile drop and one can show without difficulty that

$$H_{max} = 2\sin\frac{\gamma}{2}\left(\frac{T}{\rho g \sin\alpha}\right)^{1/2} \tag{19}$$

and that $D \to \infty$ as $H \to H_{max}$. In fact an approximate formula for D in terms of H can be readily obtained. Integrating (17) with respect to ϕ from 0 to H gives

$$p_1 H - \tfrac{1}{2}\rho g \sin\alpha H^2 - T(1-\cos\gamma) = 0 \tag{20}$$

and integrating with respect to y from 0 to D gives

$$p_1 D - \rho g \sin\alpha\beta HD - T\sin\gamma = 0 \tag{21}$$

where βHD is half the cross-section area, so that β lies between $\pi/4$ and 1. Putting $\beta = 1$ for simplicity and eliminating p_1 gives

$$D = \frac{TH\sin\gamma}{T(1-\cos\gamma) - \frac{1}{2}pg\sin\alpha H^2} \tag{22}$$

from which (18) follows.

It now appears that the curve $z = \phi(y)$ minimises the sum of surface and gravitational potential energies. The value H_{max} certainly provides an upper bound on the film thickness for which dry patches are possible but it is too crude in most cases. When the plane is vertical the free surface will be an arc of a circle and $H_{max} = \infty$.

6. CONCLUSIONS

The main purpose of this work has been to put forward a hypothesis concerning the broad structure of the flow around a stable dry patch. Calculations have been carried out to obtain values of the maximum flow rate for which a stable dry patch can exist and the results are contained in Fig. 4. The principal conclusions are: (i) The main hypothesis concerning the flow structure can only be tested in detail by comparison with experiments below the minimum wetting rate, few of which have been reported. The results seem however to be reasonable agreement with observation. (ii) In the stability of the dry patch a crucial role is played by the contact angle, and it is essential that due account should be taken, when measuring it, of evaporation, surface roughness and so on. Dynamical effects may also be important. (iii) The main restrictions on the applicability of the theory are that the *direct* mechanical effects of heat and mass transfer are small, so that boiling and turbulence produced by large temperature gradients are not accounted for. (iv) We have considered a problem in which the liquid flows downward under gravity and the dynamical effects of the external gas stream have been ignored as indicated in (iii). However in situations where the liquid is dragged horizontally by a gas stream a trivial modification should suffice, because the only important change is in the velocity profile in the film (equation (2)) and in the corresponding mass and momentum flux expressions.

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APPENDIX

The boundary value problem mentioned in Section 3 is as shown in Fig. 9. We have

$$(\nabla^2 - \lambda^2)\psi = 0 \tag{A.1}$$

in the region between the arc of a circle and the x axis,

$$\psi = 1 \quad \text{on} \quad y = 0$$

$$\frac{\partial\psi}{\partial n} = 0$$

on the curved boundary where $\partial/\partial n$ denotes differentiation along the normal. Then $f(\lambda)$ is the average value of ψ. Clearly when λ is small we have $f(\lambda) = 1 + O(\lambda^2)$.

An inspection of the magnitudes in the physical problem however indicates that λ is large near the apex. (Typically $\lambda \simeq 50$ or more.) The solution as $\lambda \to \infty$ can be obtained by a simple boundary layer argument and we have

$$\psi \sim e^{-\lambda y}$$

and so

$$f \sim \frac{2}{A\lambda}$$

where A is the area

$$= \frac{\gamma - \frac{1}{2}\sin 2\gamma}{\sin^2\lambda}.$$

We now approximate f over the whole range by the function

$$f(\lambda) = \frac{1 + k\lambda}{(1 + \frac{1}{2}k\lambda)^2}$$

where k is at our disposal. We choose k to give accuracy where we most need it, namely when λ is large, so that

$$k = 2(\lambda - \tfrac{1}{2}\sin 2\gamma)/\sin^2\lambda.$$

STABILITE D'UN RESEAU SEC SUR UNE PAROI MOUILLEE

Résumé—On développe une étude théorique de l'écoulement d'un film liquide autour d'un réseau sec stable sur une paroi chaude inclinée. Des observations ont suggéré le modèle suivant. L'écoulement est supposé non perturbé par le réseau sec, excepté dans une mince région contigüe, appelée collier, qui présente quelques caractères d'une couche limite. L'écoulement principal du film est soumis à des arguments de conservation. Il est possible de calculer l'épaisseur maximale du film pour qu'une configuration soit possible, de même que les dimensions du collier et le réseau sec, dans ces conditions. Les résultats sont comparés à ceux d'études antérieures.

DIE STABILITÄT EINES TROCKENEN FLECKS AUF EINER BENETZTEN WAND

Zusammenfassung—Es wird eine theoretische Untersuchung über die Strömung eines Flüssigkeitsfilms rings um einen stabilen trockenen Fleck auf einer benetzten, geneigten Wand beschrieben. Beobachtungen lassen das folgende Modell naheliegend erscheinen:

Es wird angenommen, daß die Strömung durch den Fleck ungestört sei bis auf ein dünnes Gebiet an seinem Rand, Kragen gennant, welches einige Eigenschaften einer Grenzschicht hat. Es besteht eine Zuordnung zur Hauptströmung des Films aufgrund von Erhaltungssätzen. Es ist möglich, die maximale Filmdicke zu berechnen, für die eine solche Konfiguration denkbar ist, ebenfalls die Abmessungen des Kragens und des trockenen Flecks in diesem Fall. Die Ergbnisse werden mit vorangegangenen Untersuchungen verglichen.

УСТОЙЧИВОСТЬ СУХОГО УЧАСТКА НА СМАЧИВАЕМОЙ СТЕНКЕ

Аннотация — Теоретически исследуется обтекание устойчивого сухого участка жидкой пленкой на наклонной смачиваемой стенке. Наблюдения позволили предложить следующую модель. Сделано предположение, что поток не возмущается сухим участком нигде за исключением узкой области около его границы, называемой воротником и обладающей некоторыми свойствами пограничного слоя. Эта область присоединяется к основному потоку пленки по соображениям, вытекающим из законов сохранения. Можно подсчитать максимальную толщину пленки, при которой такая конфигурация может существовать, а также размеры воротника и сухого участка для этих же случаев. Эти результаты сравниваются с полученными ранее.